Lösningsförslag till tentamen i Reglerteknik 150319

1. Se kursbok.

2. Avläsning ur stegsvar får

$t_p = 1 \text{sek}$

$$M \approx \frac{2}{3,5} \cdot 100 = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 0,571$$

$$t_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$$

$$\begin{cases} \omega_0 = 3,2 \text{ rad/s} \\ \zeta = 0,175 \end{cases}$$

$$G(s) = \frac{K \cdot \omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \quad \text{där} \quad k = \frac{\Delta y}{\Delta u} = \frac{3,5}{3,5} = 1$$

3.

a) $$H(j\omega) = \frac{U_{ut}}{U_{in}} = \frac{R_2}{R_2 + \frac{1}{j\omega C_1} /\!/ R_1} = \frac{R_2}{R_2 + \frac{R_1}{1 + j\omega C_1 R_1}} =$$

$$= \frac{R_2(1 + j\omega C_1 R_1)}{R_1 + R_2 + j\omega C_1 R_1 R_2}$$

b) $$|H(j\omega)| = \frac{R_2\sqrt{1 + (\omega C_1 R_1)^2}}{\sqrt{(R_1+R_2)^2 + (\omega C_1 R_1 R_2)^2}}$$

amplitudfkn.

fasfunktion: $$\arg\{H(j\omega)\} = \arctan(\omega C_1 R_1) - \arctan\left(\frac{\omega C_1 R_1 R_2}{R_1 + R_2}\right)$$

c) Se bifogat Bodediagram

d) $$U_{ut}(t) = 10 \cdot |H(j \cdot 1)| \sin(1 \cdot t + \arg\{H(j1)\})$$

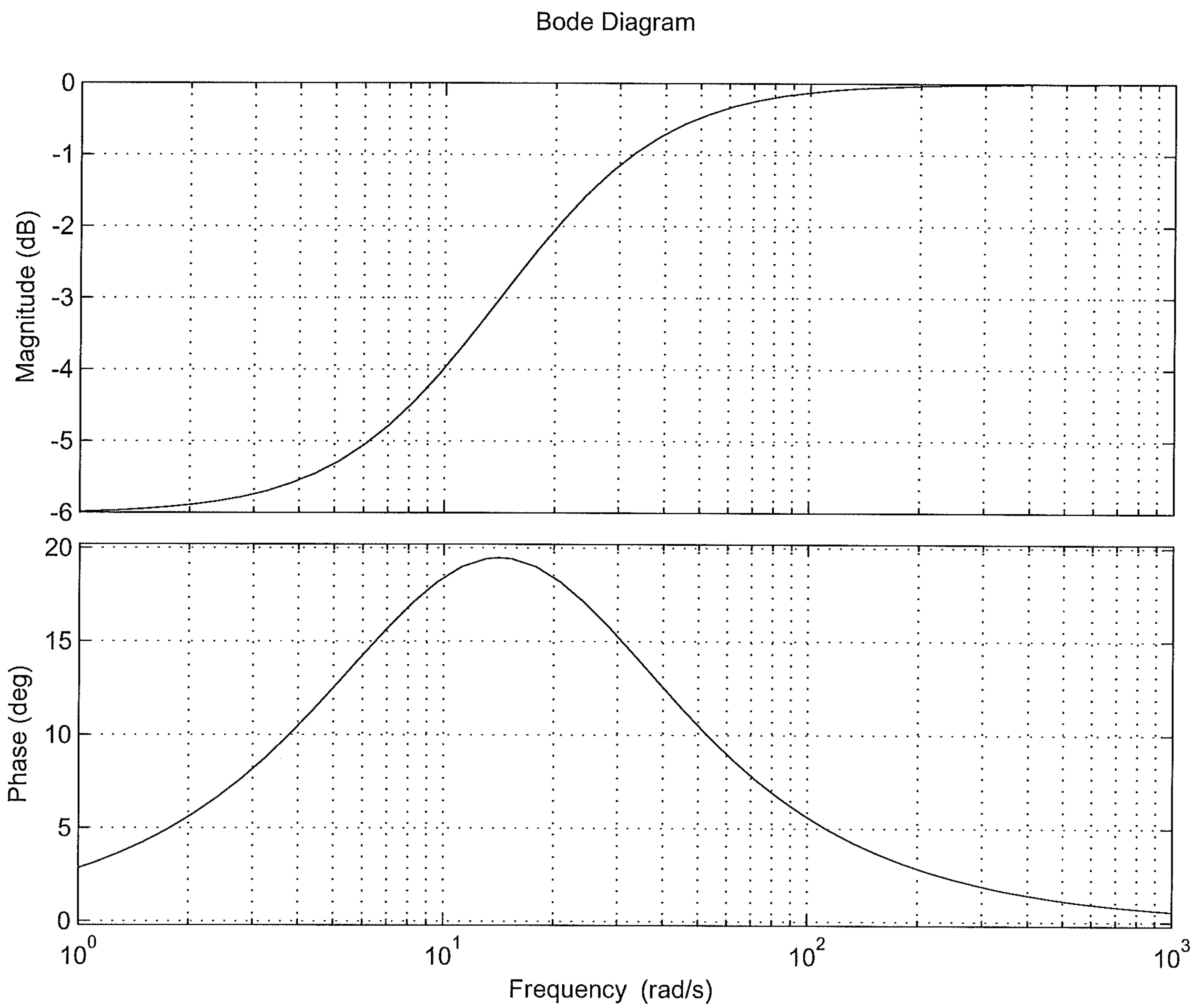
3c

$$H(j\omega) = \frac{10000\,(1 + 0{,}1 j\omega)}{20000 + j\omega \cdot 1000}$$

$\leftarrow \omega_b = 10$ rad/s

$\nwarrow \omega_b = 20$

Intressant intervall att titta på: 1 rad/s – 200 rad/s



4.

$$H(s) = \frac{4}{s+2} = \frac{2}{0{,}5s+1} \longrightarrow H_p(z) = \frac{K(1-e^{-h/T})z^{-1}}{(1-e^{-h/T} z^{-1})} \approx$$

grad P = 1
grad C = 0, grad D = 0

$$\approx \frac{2\cdot(1-e^{-1})z^{-1}}{(1-e^{-1} z^{-1})} \approx \frac{1{,}26 z^{-1}}{1-0{,}37 z^{-1}} = \frac{B}{A}$$

$$P = AC + BD$$

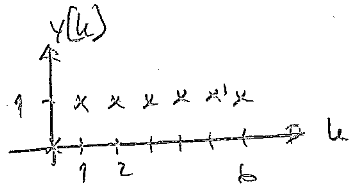
$$1 = (1-0{,}37z^{-1})\cdot 1 + 1{,}26 z^{-1}\cdot d_0$$

$$d_0 \approx 0{,}29 \qquad K_r = \frac{P(1)}{B(1)} = \frac{1}{1{,}26} \approx 0{,}79$$

$$\frac{Y}{R} = \frac{K_r \cdot B}{P} = \frac{0{,}79\cdot 1{,}26 z^{-1}}{1} = z^{-1} \qquad y(k) = r(k-1)$$

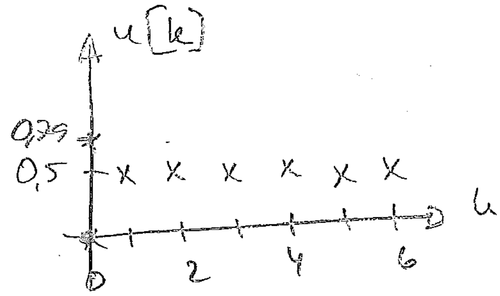
a)

k	y	r
0	0	0
1	1	1
2	1	1
3	1	1
4	1	1
⋮	⋮	⋮



b) $(R(z)\cdot K_r - D(z)\cdot Y(z))\frac{1}{C(z)} = U(z) \implies u[k] = 0{,}79\cdot r[k] - 0{,}29\, y(k)$

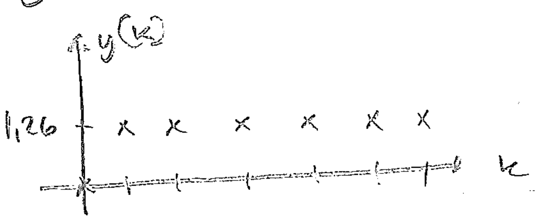
k	u[k]	y[k]	r[k]
0	0,79	0	1
1	0,5	1	1
2	0,5	1	1
3	0,5	1	1
4	0,5	1	1
⋮	⋮	⋮	⋮



c)

$$\frac{Y}{V} = \frac{\frac{B}{A}}{1+\frac{B}{A}\cdot\frac{D}{C}} = \frac{BC}{P} = \frac{1{,}26 z^{-1}}{1}$$

$$y(k) = 1{,}26\cdot v[k-1]$$



klarar inte stegstörningar ⇒ kvarstående fel.

$$e(\infty) = -1{,}26$$

5.

Kretsöverföringen: $G_R \cdot G_p = 8\left(1+\frac{1}{100s}\right) \cdot \frac{2e^{-s}}{(1+20s)(1+10s)}$

$$= 8\left(\frac{100s+1}{100s}\right) \cdot \frac{2e^{-s}}{(1+20s)(1+10s)}$$

$$|G_R \cdot G_p| = 16 \cdot \frac{\sqrt{(100\omega)^2+1}}{100\omega\sqrt{1+400\omega^2}\sqrt{1+100\omega^2}}$$

$$\arg\{G_R \cdot G_p\} = -90^\circ + \arctan(100\omega) - \omega \cdot \frac{180}{\pi} - \arctan(20\omega) - \arctan(10\omega)$$

ω	$\arg\{G_R \cdot G_p\}$	$\lvert G_R \cdot G_p \rvert$
0,1	-120°	5,1
0,2	-154°	1,74
0,27	-167°	1,01
0,3	-171°	0,83
0,5	-193°	0,31
1	-229°	0.08
0,37	-180°	0,56

a)

$\varphi_m \approx 13^\circ$

$A_m \approx \frac{1}{0,56} \approx 1,8$ ggr

b) Kvarstående fel. vid laststörningssteg $= 0$

— " — vid enhetsramp $= \frac{1}{|G_R \cdot G_p(0)|}$

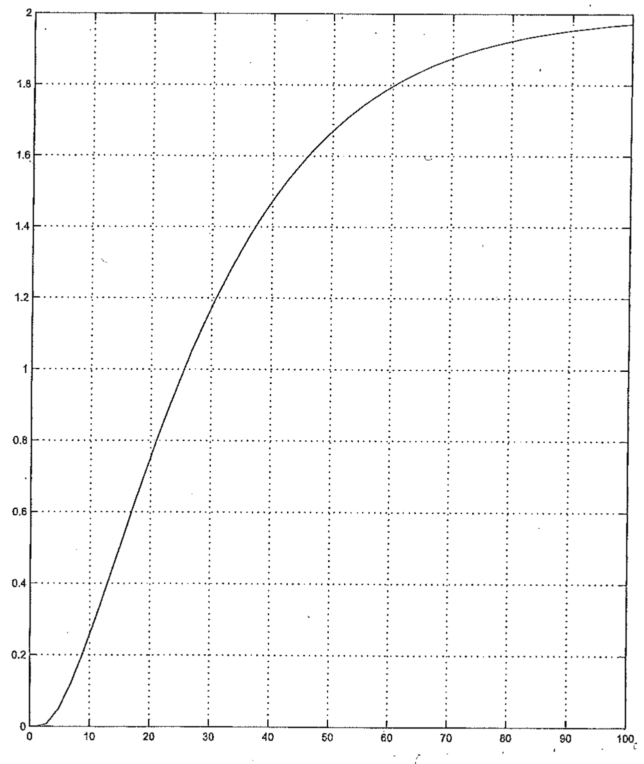
$$e_{ss} = \frac{1}{K_1} = \frac{1}{0,16} \approx 6,25$$

$$G_R \cdot G_p(0) = \frac{16/100}{s} = \frac{K_1}{s}$$

5c

Processen $G_p(s) = \dfrac{2e^{-1s}}{(1+10s)(1+20s)}$

Stegsvaret ser ut enligt nedan:



Det finns ett antal tumregelmetoder som baseras på stegsvar som kan användas. Även Ziegler-Nichols självsvängn. metod skulle kunna vara möjlig.

Jag använder Lambdametoden $\Longrightarrow \begin{cases} L = 1 \text{ sek} \\ T = 10+20 \text{ sek} = 30 \text{ s} \end{cases}$

$M = \max\{1, 30\} = 30$, välj $p = 2$

$\lambda = p \cdot M = 2 \cdot 30 = 60$

PI-reg: $K = \dfrac{T}{K_p(\lambda + L)} = \dfrac{30}{2(60+1)} = \dfrac{30}{122} \simeq 0{,}25$

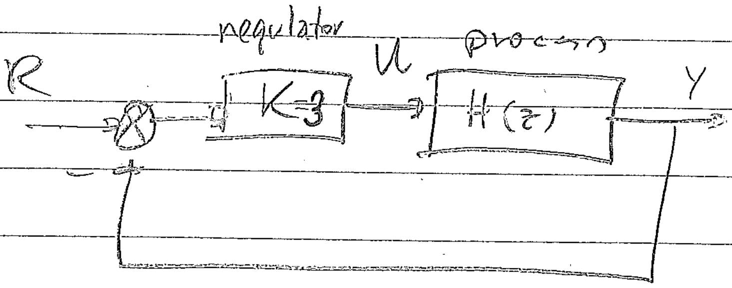
$T_i = T = 30$ sek

6.

process

$$G(s) = \frac{1}{1+10s} \xrightarrow{\{\text{F.S. sid 12}\}} \text{Diskret process} \quad H(z) = \frac{K\left(1-e^{-h/T}\right)z^{-1}}{1-e^{-h/T}\cdot z^{-1}} =$$

$$= \frac{1\cdot\left(1-e^{-h/10}\right)z^{-1}}{1-e^{-h/10}z^{-1}}$$



a) karakt. ekv.

$$1+\frac{3\cdot 1\left(1-e^{-h/10}\right)z^{-1}}{1-e^{-h/10}z^{-1}} = 0$$

$$1-e^{-h/10}z^{-1}+3\cdot 1\left(1-e^{-h/10}\right)z^{-1} = 0$$

$$z = e^{-h/10}-3+3e^{-h/10}$$

stabilitetsgräns $|z|=1 \longrightarrow h \approx 6,9$ sek

$h < 6,9$ sek

b) $h = \frac{h_{max}}{3} \approx 2,3$ sek då $K=3$

$$H(z) = \frac{0,206\,z^{-1}}{1-0,795\,z^{-1}} \qquad H_{TOT}(z) = \frac{K\cdot H(z)}{1+K\cdot H(z)}$$

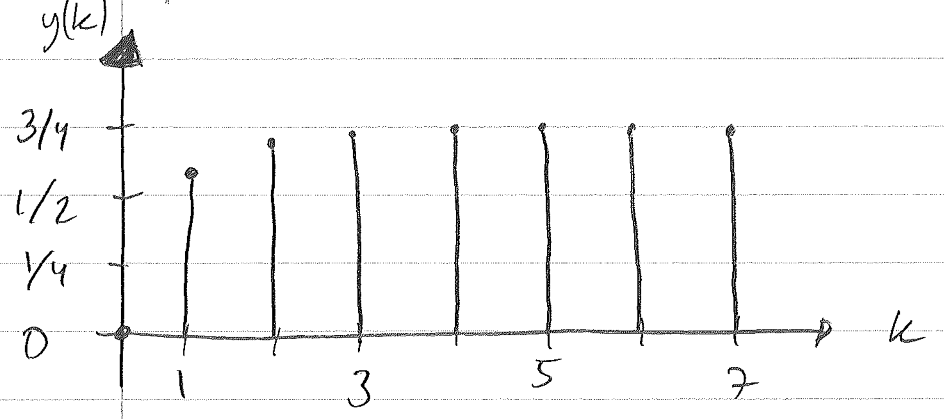
$$H_{TOT}(z) = \frac{\dfrac{3\cdot 0,206\,z^{-1}}{1-0,795\,z^{-1}}}{1+\dfrac{3\cdot 0,206\,z^{-1}}{1-0,795\,z^{-1}}} = \frac{0,618\,z^{-1}}{1-0,795\,z^{-1}+0,618\,z^{-1}} =$$

$$H_{TOT}(z) = \frac{0,618\cdot z^{-1}}{1-0,176\,z^{-1}} = \frac{Y(z)}{R(z)}$$

Differensekvation: $y(k) = 0,176\,y(k-1) + 0,618\,r(k-1)$

6b forts.

k	y(k)	= 0,176 y(k-1)	+ 0,618 r(k-1)
0	0	0	0
1	0,618	0	0,618
2	0,727	0,109	0.618
3	0,746	0,128	0.618
4	0,749	0.131	0.618
5	0.750	0,132	0,618
6	0,75	0.132	0.618
7	0,75	0,132	0,618



c) $e(\infty) = y(k) - r(k) = \frac{1}{4}$
när $k \to \infty$

d) Kretsöverföring: $H_{reg}(z) \cdot H(z)$ $\quad 3 \cdot \dfrac{0{,}206 z^{-1}}{1 - 0{,}794 z^{-1}} = \dfrac{0{,}618 z^{-1}}{1 - 0{,}794 z^{-1}} = \dfrac{0{,}618}{z - 0{,}794}$

Sätt $z = e^{j\omega h}$ för att ta fram diskret amplitud- och fasfunktion!

$$H_R \cdot H_p(e^{j\omega h}) = \frac{0{,}618}{e^{j\omega h} - 0{,}794} = \frac{0{,}618}{\cos\omega h + j\sin\omega h - 0{,}794}$$

$$|H_R \cdot H_p(e^{j\omega h})| = \frac{0{,}618}{\sqrt{(\cos\omega h - 0{,}794)^2 + \sin^2\omega h}} \approx \frac{0{,}618}{\sqrt{1{,}63 - 1{,}588\cos\omega h}}$$

$$\arg\{H_R \cdot H_p\} = -\arctan\left(\frac{\sin\omega h}{\cos\omega h - 0{,}794}\right) \pm n \cdot 180°$$

7. Enligt Amplitudkurvan fås en

a) struktur: $\frac{K}{s} \cdot \frac{1}{1+\frac{s}{\omega_b}}$

Detta bekräftas enligt faskurvan.

$\omega_b \approx 10$ rad/s ri har en fasvridning på $-(90^\circ + 45^\circ) = -135^\circ$

Ur LF-asymptoten fås $\frac{K}{\omega} = 1$ vid $\omega = 4$.

HF-asymptot luter -40dB/dek.

$$G(s) = \frac{4}{s} \cdot \frac{1}{1+\frac{s}{10}}$$

b) Vår karakt. ekv blir ifall vi har en PI-reg.

$$1 + G_R \cdot G_p(s) = 0$$

$$1 + K\left(\frac{T_i s + 1}{T_i s}\right) \cdot \frac{4}{s \cdot (1+0{,}1s)} = 0$$

$$T_i s^2 \cdot (1+0{,}1s) + 4K(T_i s + 1) = 0$$

$$0{,}1 T_i s^3 + T_i s^2 + 4KT_i \cdot s + 4K = 0$$

Routh - Hurwitz tabell

s^3	$0{,}1T_i$	$4KT_i$	0
s^2	T_i	$4K$	0
s^1	$\frac{4KT_i^2 - 4KT_i \cdot 0{,}1}{T_i}$	0	
s^0	$4K$		

$\to T_i > 0$

$\to T_i > 0$

$\to 4KT_i(T_i - 0{,}1)$

$\to K > 0$

Fås från problemformulering
$\begin{cases} T_i > 0 \\ K > 0 \end{cases}$

$T_i > 0$ oberoende av K om denna är positiv.